

МОСКОВСКИЙ ФИЗИКО-ТЕХНИЧЕСКИЙ ИНСТИТУТ

ОЛИМПИАДА "ФИЗТЕХ" ПО МАТЕМАТИКЕ

11 класс

ВАРИАНТ 8

ШИФР

Бланк задания должен быть вложен в раб
Работы без вложенного задания не проверяются.

- ✓ 1. [3 балла] Найдите количество восьмизначных чисел, произведение цифр каждого из которых равно 64827. Ответ необходимо представить в виде целого числа.
- ✓ 2. [5 баллов] Решите уравнение $\cos 7x + \cos 3x + \sin 7x - \sin 3x + \sqrt{2} \cos 4x = 0$.
- ✓ 3. [5 баллов] Решите систему уравнений

$$\begin{cases} \left(-\frac{x^7}{y}\right)^{\ln(-y)} = x^{2\ln(xy^2)}, \\ y^2 + 2xy - 3x^2 + 12x + 4y = 0. \end{cases}$$

4. [5 баллов] Сфера с центром O вписана в трёхгранный угол с вершиной S и касается его граней в точках K, L, M (все плоские углы трёхгранного угла различны). Найдите угол KSO и площадь сечения данного трёхгранного угла плоскостью KLM , если известно, что площади сечений трёхгранного угла плоскостями, касающимися сферы и перпендикулярными прямой SO , равны 9 и 16.
- ✓ 5. [5 баллов] Найдите все значения параметра a , при которых система

$$\begin{cases} |x + y + 8| + |x - y + 8| = 16, \\ (|x| - 8)^2 + (|y| - 15)^2 = a \end{cases}$$

имеет ровно два решения.

- ✓ 6. [6 баллов] а) Две окружности одинакового радиуса 17 пересекаются в точках A и B . На первой окружности выбрана точка C , а на второй – точка D . Оказалось, что точка B лежит на отрезке CD , а $\angle CAD = 90^\circ$. На перпендикуляре к CD , проходящем через точку B , выбрана точка F так, что $BF = BD$ (точки A и F расположены по разные стороны от прямой CD). Найдите длину отрезка CF .
- б) Пусть дополнительно известно, что $BC = 16$. Найдите площадь треугольника ACF .
7. [6 баллов] Найдите количество пар целых чисел (x, y) , удовлетворяющих системе неравенств

$$\begin{cases} y > 3^x + 4 \cdot 3^{28} \\ y \leq 93 + 3(3^{27} - 1)x \end{cases}$$

Ответ должен быть представлен в виде алгебраической суммы не более двух слагаемых.

UNITED STATES DEPARTMENT OF THE INTERIOR
BUREAU OF LAND MANAGEMENT

WATER RESOURCES DIVISION

Report of Progress and Accomplishments
for the Year Ending December 31, 1964

The following report was prepared by the Bureau of Land Management, Department of the Interior, for the purpose of providing information on the progress and accomplishments of the Bureau during the year ending December 31, 1964. The report is organized into three main sections: (1) General Information, (2) Major Projects, and (3) Financial Summary.

The first section, General Information, contains information on the Bureau's organization, personnel, and activities during the year. The second section, Major Projects, contains information on the progress and accomplishments of the Bureau's major projects during the year. The third section, Financial Summary, contains information on the Bureau's financial activities during the year.

The following is a summary of the progress and accomplishments of the Bureau during the year ending December 31, 1964. The Bureau has completed a number of major projects, including the construction of a new water control structure at the mouth of the Colorado River, the construction of a new water control structure at the mouth of the Colorado River, and the construction of a new water control structure at the mouth of the Colorado River.

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ПИСЬМЕННАЯ РАБОТА

1

$$64827 = 27609 \cdot 3 = 7203 \cdot 3^2 = \cancel{4029} \cdot 1029 \cdot 7 \cdot 3^2 = 343 \cdot 7 \cdot 3^3 =$$

$$= \frac{7203}{3} \cdot 3 = 2401 \cdot 3$$

$7^2 > 10 \Rightarrow$ в шестке по 4 цифры 7

3 входит в шестке 3 или 9, ведь $3^3 > 10$, ~~2~~

$$3 \cdot 7 > 10$$

1) 4 цифры 7, 3 цифры 3 и 1 цифра 1

вариантов расстановки

$$\frac{8!}{4! \cdot 3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{6} = 8 \cdot 7 \cdot 5$$

2) 4 цифры 7, 1 "3", 1 "9" и 2 "1"

вариантов

$$\frac{8!}{4! \cdot 2} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{2} = 3 \cdot 8 \cdot 7 \cdot 5$$

$$\text{Всего вариантов } 8 \cdot 7 \cdot 5 + 3 \cdot 8 \cdot 7 \cdot 5 = 4 \cdot 8 \cdot 7 \cdot 5 = 2056$$

$$= 1120$$

Ответ: 1120

$$\begin{array}{r} 7203 \\ 102908 \\ 7 \\ \hline 7203 \end{array}$$

$$\begin{array}{r} 7029 \overline{) 3} \\ 9 \\ \hline 72 \end{array}$$

$$(y+3x)(y-x+9)$$

$$2 \sin\left(2x + \frac{\pi}{4}\right) \cos 5x$$

$$\frac{3}{2} x - \frac{\pi}{4} = \pi n + \frac{\pi}{2}$$

$$2 \cos 5x \cos 2x + 2 \sin 2x \cos 5x \neq \sqrt{2} \cos 4x$$

$$\frac{3}{2} x = \pi n + \frac{3}{4} \pi$$

$$2 \sin 2x \cos 5x + \cos 4x = 0$$

$$\begin{aligned} x &= \frac{2}{3} \pi n + \frac{2}{3} \pi \\ \frac{3}{8} + \frac{2}{7} &= \frac{2}{28} \end{aligned}$$

$$\cos 7x + \cos 3x + \sin 7x - \sin 3x + \sqrt{2} \cos 4x$$

$$\frac{1}{3} \ln 3 = \frac{2}{3} \ln 9$$

$$2 \sin^2 2x + 2 \sin 2x \cos 5x = 0$$

$$2 \cos 5x \cos 2x + 2 \sin 5x \cos 2x + \sqrt{2} \cos 4x$$

$$2 \sqrt{2} \cos 2x \cos \left(5x + \frac{\pi}{4}\right) + 2 \cos^2 2x - 7$$

$$\cos 2x$$

$$\sin\left(7x + \frac{\pi}{4}\right) = \sin\left(5x - \frac{\pi}{4}\right) + \cos 4x = 0$$

$$2 \sin\left(2x + \frac{\pi}{4}\right) \cos 5x + \cos 4x = 0$$

$$\sqrt{2} (\sin 2x + \cos 2x) \cos 5x + (\cos^2 2x - \sin^2 2x) = 0$$

$$(\sin 2x + \cos 2x) (\sqrt{2} \cos 5x + \cos 2x - \sin 2x) = 0$$

$$\sin 2x$$

$$\sqrt{2} 2 \sin\left(2x + \frac{\pi}{4}\right) (\cos 5x + \cos\left(2x + \frac{\pi}{4}\right)) = 0$$

$$\sin\left(2x + \frac{\pi}{4}\right) \cos\left(\frac{3}{2}x + \frac{\pi}{8}\right) \cos\left(1,5x - \frac{\pi}{8}\right) = 0$$

ПИСЬМЕННАЯ РАБОТА

2

$$\cos 2x + \cos 3x + \sin 7x - \sin 3x + \sqrt{2} \cos 4x = 0$$

$$\cos 7x + \sin 7x = \sqrt{2} \sin\left(7x + \frac{\pi}{4}\right)$$

$$\cos 3x - \sin 3x = \sqrt{2} \cos\left(3x + \frac{\pi}{4}\right) - \sqrt{2} \sin\left(3x - \frac{\pi}{4}\right)$$

$$\cos 4x + \sin\left(7x + \frac{\pi}{4}\right) + \cos\left(3x + \frac{\pi}{4}\right) = 0$$

$$\cos 4x + \sin\left(7x + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4} - 3x\right) = 0$$

$$\sin\left(7x + \frac{\pi}{4}\right) - \sin\left(3x - \frac{\pi}{4}\right) + \cos 4x = 0$$

$$2 \sin\left(2x + \frac{\pi}{4}\right) \cos(5x) + \cos 4x = 0$$

$$\cos 4x = (\cos x - \sin 2x)(\cos 2x + \sin 2x) = (\cos(2x) - \sin 3x) - \sqrt{2} \sin\left(2x + \frac{\pi}{4}\right)$$

$$\sqrt{2} \sin\left(2x + \frac{\pi}{4}\right) (\sqrt{2} \cos(5x) + \cos 2x - \sin 2x) = 0$$

$$\sin\left(2x + \frac{\pi}{4}\right) (\cos 5x + \cos\left(2x + \frac{\pi}{4}\right)) = 0$$

$$\sin\left(2x + \frac{\pi}{4}\right) \cos\left(3,5x + \frac{\pi}{8}\right) \cos\left(1,5x - \frac{\pi}{4}\right) = 0$$

$$\sin\left(2x + \frac{\pi}{4}\right) = 0$$

$$x = \frac{7\pi}{2} - \frac{\pi}{8}$$

$$\cos\left(3,5x + \frac{\pi}{8}\right)$$

$$x = \frac{2}{7}\pi - \frac{3}{28}\pi$$

$$\cos\left(1,5x - \frac{\pi}{4}\right) = 0$$

$$x = \frac{2}{3}\pi + \frac{\pi}{2}$$

$$\text{Омбери: } \begin{cases} x = \frac{\pi n}{2} - \frac{\pi}{8} & n \in \mathbb{Z} \\ x = \frac{2}{3} \pi n - \frac{3}{28} \pi & n \in \mathbb{Z} \\ x = \frac{2}{3} \pi n + \frac{\pi}{2} & n \in \mathbb{Z} \end{cases}$$

$$\begin{cases} \left(-\frac{x^7}{y} \right)^{\ln(-y)} = x^{2 \ln(xy^2)} \\ y^2 + 2xy - 3x^2 + 12x + 4y = 0 \end{cases}$$

$$0.2.3 \quad y \leftarrow \in (-\infty; 0)$$

$$x \rightarrow x \in (0; +\infty)$$

$$\begin{cases} \left(-\frac{x^7}{y} \right)^{\ln(-y)} = x^{2 \ln(xy^2)} \\ (y+3x)(y-x+4) = 0 \end{cases}$$

$$1) \quad y = -3x$$

$$\left(\frac{x^6}{3} \right)^{\ln 3x} = x^{2 \ln 9x^3}$$

$$e^{\ln \frac{x^6}{3} \cdot \ln 3x} = e^{\ln x^2 \ln 9x^3}$$

$$\ln x = \frac{1}{3}$$

$$6 \ln x - \ln 3 \ln 3 = 2 \ln x (2 \ln 3 + 3 \ln x)$$

$$5 \ln 3 \cdot \ln x + 6 (\ln x)^2 - (\ln 3)^2 = 4 \ln 3 \cdot \ln x + 6 (\ln x)^2$$

$$\ln 3 \ln x - (\ln 3)^2 = 0$$

$$\ln x = \ln 3$$

$$x = 3$$

$$y = -3$$

$$2) \quad y = \frac{x-y}{\ln(y-x)} = x^{2 \ln(x(y-x)^2)} \quad x \in (0; 4)$$

$$(7 \ln x - \ln(y-x)) \ln(y-x) = 2 \ln x (\ln x + 2 \ln(y-x))$$

$$-3 (\ln x) \cdot \ln(y-x) + (\ln(y-x))^2 + 2 (\ln x)^2 = 0$$

ПИСЬМЕННАЯ РАБОТА

$$(\ln x - \ln(4-x)) \quad (\ln x - 2\ln(4-x)) = 0$$

$$1) \ln x = \ln(4-x)$$

$$x = 2$$

$$y = -2$$

$$2) \ln x = 2\ln(4-x)$$

$$x^2 = 16 - 8x + x^2$$

$$x^2 - 8x + 16 = 0$$

$$D = 64 - 64 = 0$$

$$x = \frac{8 \pm \sqrt{0}}{2} = 4$$

$$\text{при } x = \frac{8 + \sqrt{0}}{2} \quad x > 4 \Rightarrow$$

$$\Rightarrow x = \frac{8 - \sqrt{0}}{2} \quad y = \frac{7 - \sqrt{0}}{2}$$

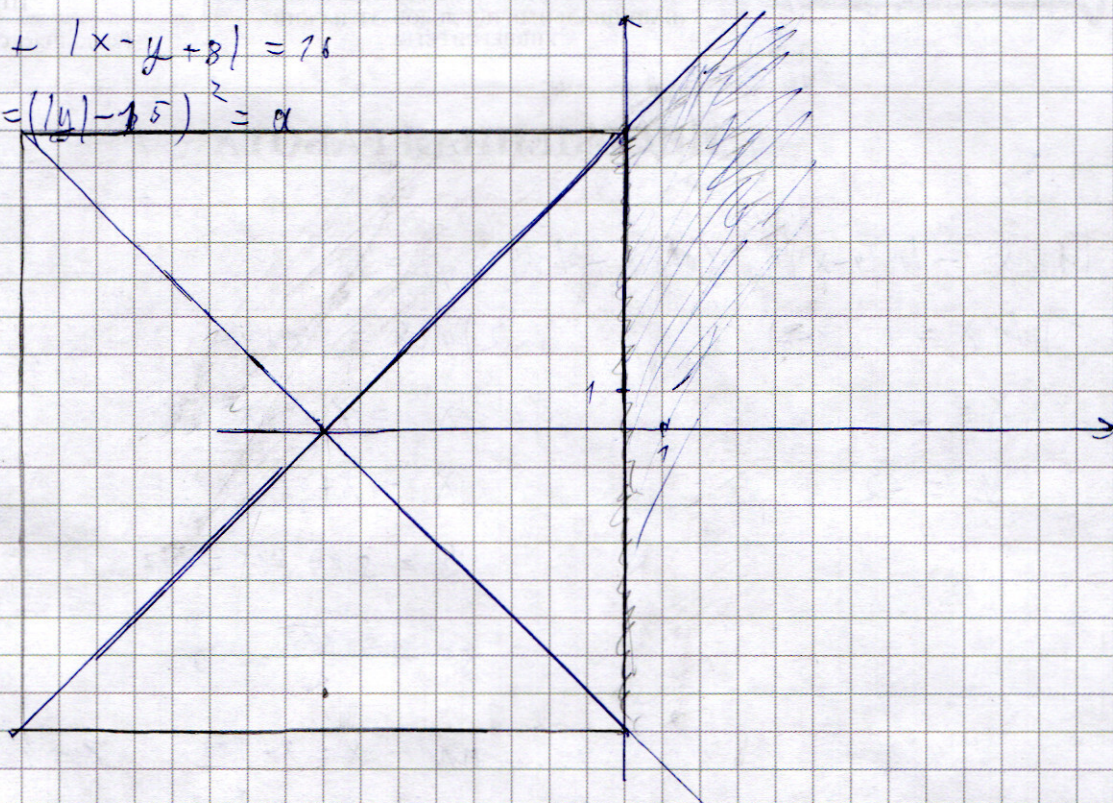
$$\text{Ответ: } (3; -3) \quad (2; -2) \quad \left(\frac{8 - \sqrt{0}}{2}; \frac{7 - \sqrt{0}}{2} \right)$$

$$|x+y+8| + |x-y+8| = 16$$

$$(x-8)^2 = (y-15)^2 = a$$

$$2x+16=-16$$

$$x=-16$$



$$\begin{array}{r} 64 \\ + 49 \\ \hline 113 \end{array}$$

$$\begin{array}{r} 23 \\ \times 23 \\ \hline 69 \\ 46 \\ \hline 529 \end{array}$$

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$$

$$\begin{array}{r} 528 \\ + 593 \\ \hline 1121 \end{array}$$

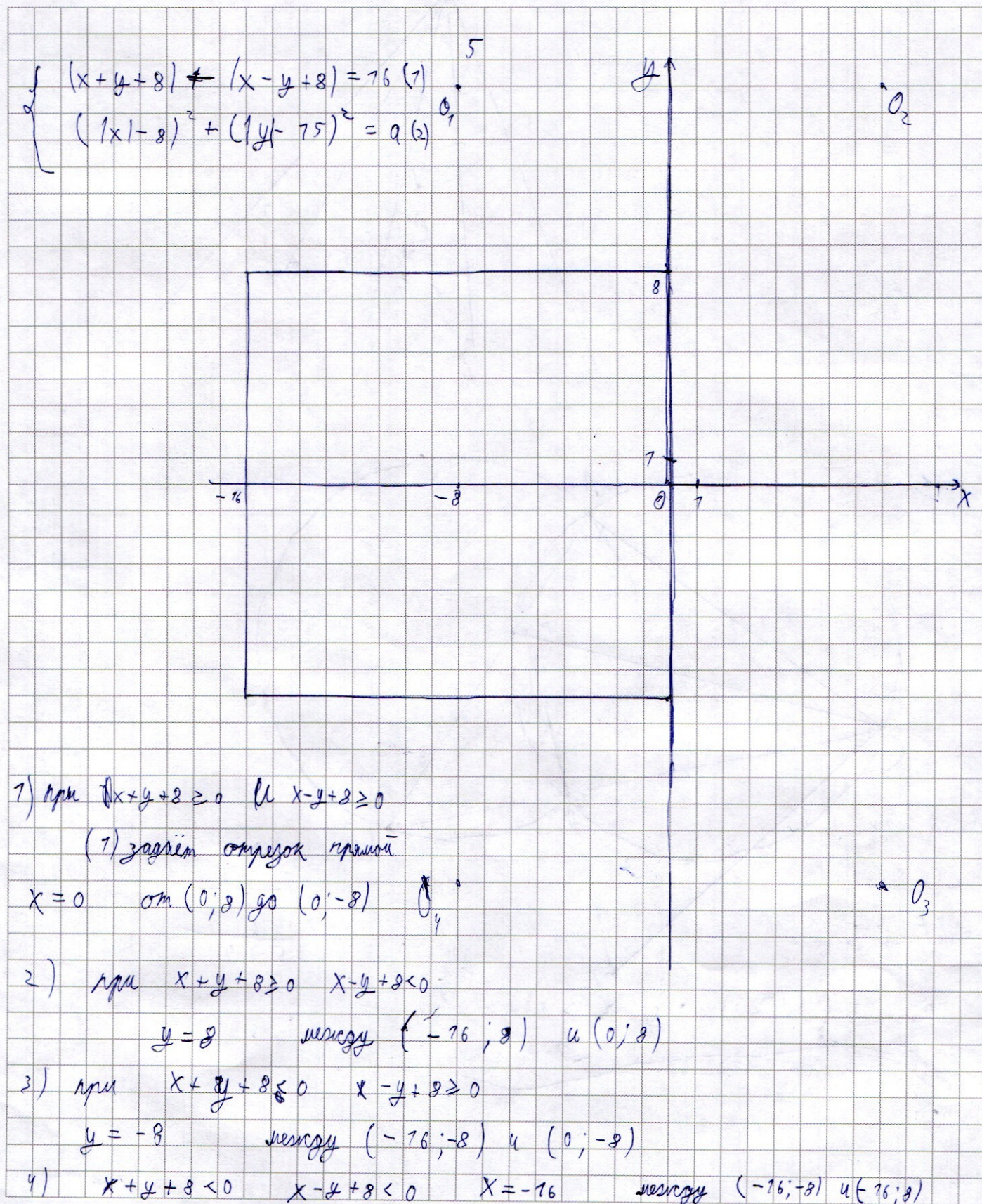
$$y > 3^x + 4 \cdot 3^{28}$$

$$y \leq 93 + 3(3^{27} - 1)x$$

$$93 + 3^{28}x - 3x > 3^x + 4 \cdot 3^{28}$$

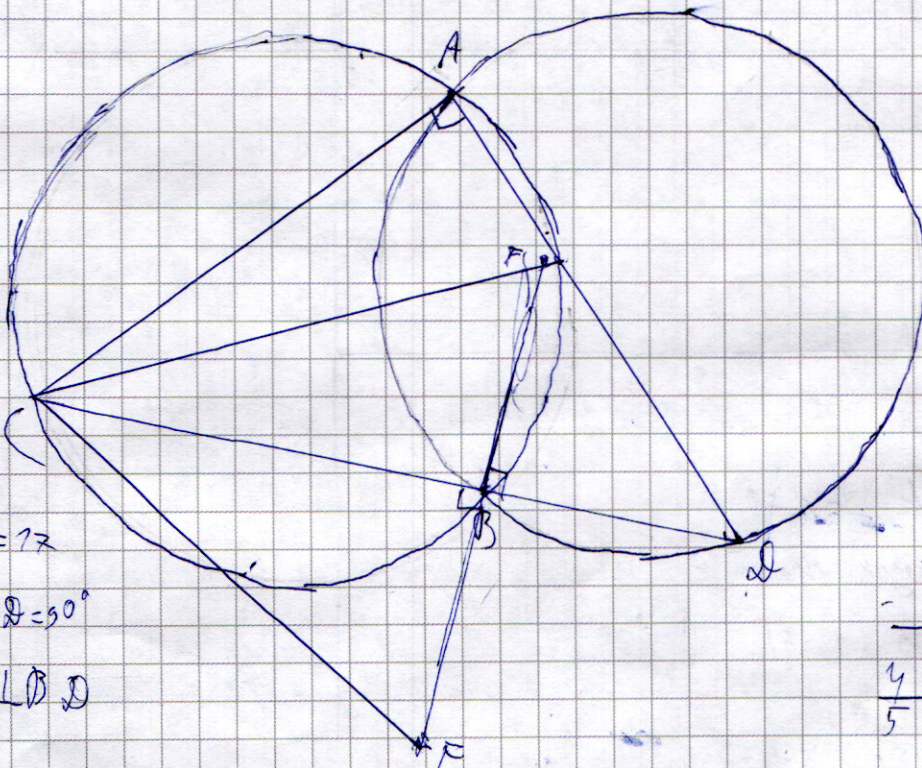
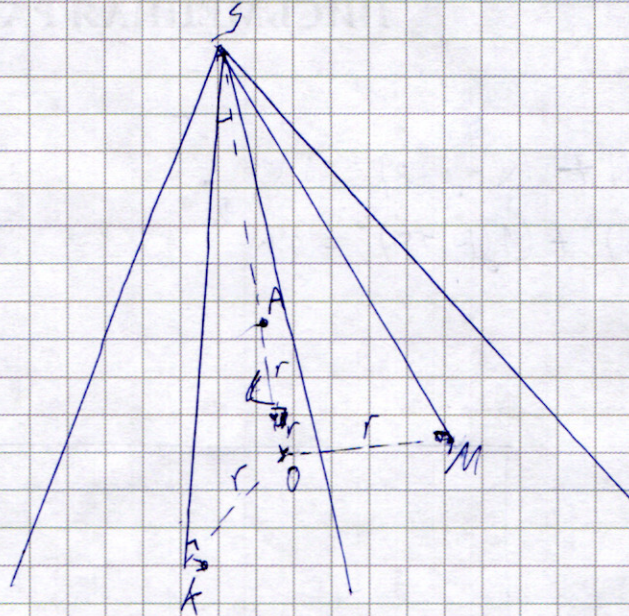
$$3^{28} + 90 > 3 + 4 \cdot 3^{28}$$

ПИСЬМЕННАЯ РАБОТА



7

$$\angle KSO = ?$$



$$R_1 = R_2 = 72$$

$$\angle CAD = 90^\circ$$

$$BF \perp CD$$

$$\begin{array}{r} 23 \\ - 23 \\ \hline 69 \\ 46 \\ \hline 529 \end{array}$$

$$93 + 3(3^{27} - 1) \times 3 \times 3 > 3^X \times 9 \cdot 3^{28}$$

$$93 + 3(3^{27} - 1) \times 3 \times 3 > 3^X \times 9 \cdot 3^{28}$$

$$93 + 3(3^{27} - 1) \times 3 \times 3 > 3^X \times 9 \cdot 3^{28}$$

ПИСЬМЕННАЯ РАБОТА

(2) задан 4 окружности с центрами $(-8; 15)$; $(8; 15)$; $(8; -15)$; $(-8; -15)$ и радиусами $\sqrt{a}$ при $a > 0$ при $a = 0$ 4 точки, не совпадающие с $\square$ квадратом при $a < 0$ не имеет решений

при $a \in (0; 49)$ окружности и квадрат не пересекаются

при $a = 49$ есть 2 решения $(-8; 8)$ и $(-8; -8)$

при $a \in (49; \sqrt{773})$ 4 решения

при $\sqrt{a} \in (\sqrt{773}; \sqrt{769})$ 2 решения

при $a > \sqrt{769}$ 0 решений

при $\sqrt{a} = \sqrt{769}$ есть 2 решения $(-16; 8)$; $(-16; -8)$

при $\sqrt{a} > \sqrt{769}$ 0 решений

Ответ: при $a = \{49; \sqrt{769}\}$

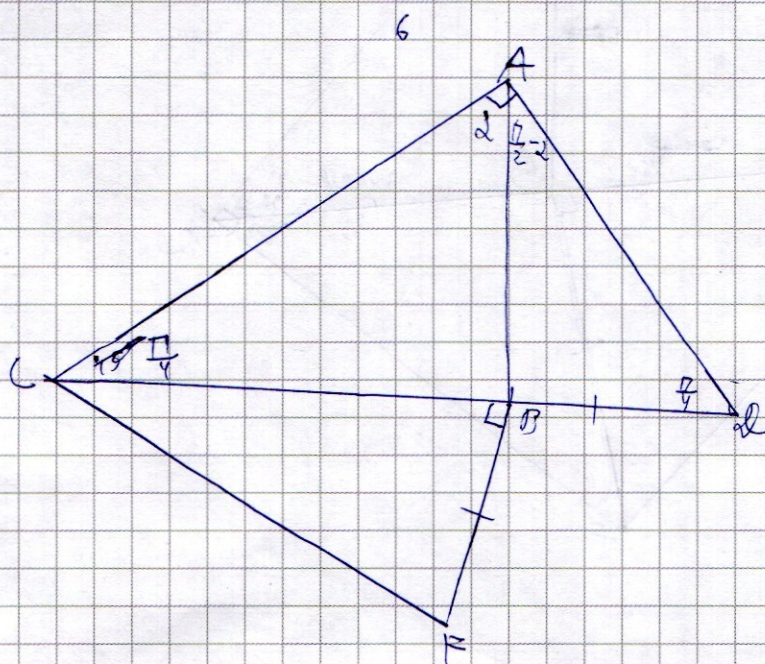


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ПИСЬМЕННАЯ РАБОТА

a)



$$\angle ACD + \angle ADC = \frac{\pi}{2} \Rightarrow \sin \angle ACD = \cos \angle ADC$$

по теореме синусов для $\triangle ACB$ $AB = 34 \sin \angle ACB$

gila $\triangle A B D$ $AB = 34 \sin \angle A D B$

$$\sin \angle ACB = \sin \angle ADB = \sin \left(\frac{\pi}{2} - \angle ACB \right)$$

$$\angle ACB = \angle ADB = 90^\circ$$

$$\overline{f_{\text{Ly}\alpha}} \subset (AB) = 2 \text{ } \odot, \text{ margin} \subset (AD) = \frac{1}{3} - 2$$

по теореме синусов $CB = 34 \sin 2$, $CD = 34 \cos 2 = DF$

$$\angle (BF = \frac{1}{2}) \Rightarrow CF = \sqrt{CB^2 + BF^2} = 34.$$

Amber $(F = 34)$

$$\text{ii) } BC = 39 \sin 2 = 76$$

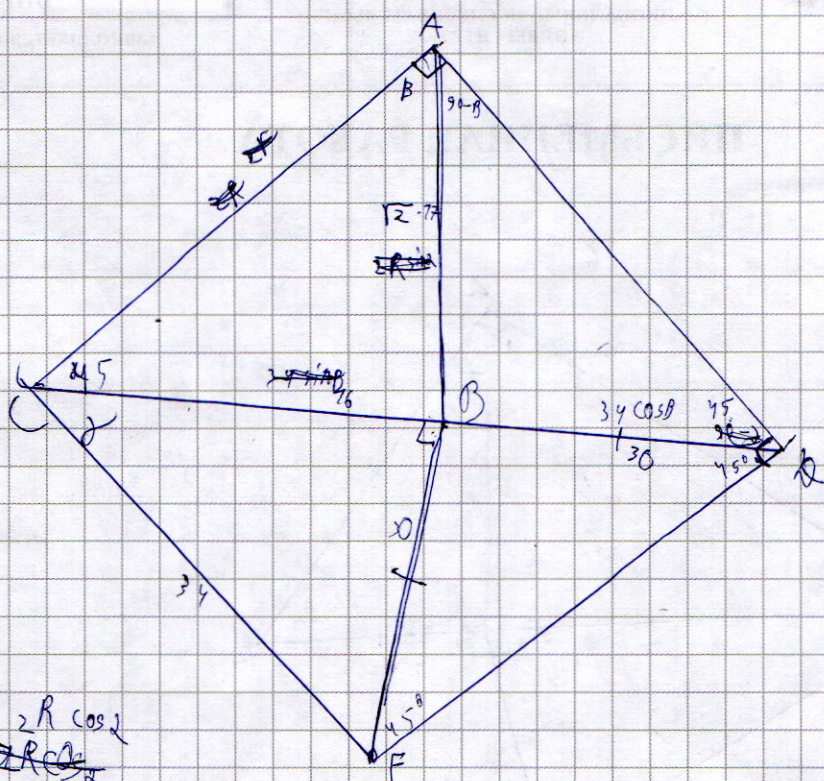
$$9/12 = 8$$

$$\cos 2 = \frac{75}{17}$$

~~$BQ = 34 \cos 2 = 20$~~ $\angle ABC = \frac{3}{4}\pi - 2$

$$\frac{a}{\sin \alpha} = 2R$$

~~R = a~~



$$2R \sin \alpha = \frac{2R \cos \alpha}{\sin \alpha}$$

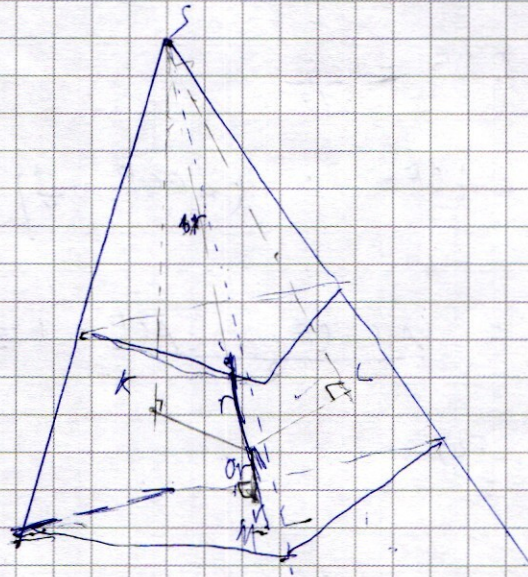
$$\sin \alpha = \frac{1}{2}$$

~~30~~ 30

$$\sin \beta = \frac{26}{34}$$

$$\sin \gamma = \frac{30}{34}$$

ПИСЬМЕННАЯ РАБОТА



$$AC = 34 \sin \angle ABC = 34 \sin\left(\frac{\pi}{4} + \alpha\right) = \cancel{24\sqrt{2}} \cdot 17\sqrt{2} \cdot \frac{23}{72} = 23\sqrt{2}$$

$$\sin \angle CFB = \frac{CB}{CF} = \frac{8}{17}$$

$$\sin \angle BCF = \frac{15}{17}$$

$$\sin \angle ACF = \sin\left(\angle BCF + \frac{\pi}{4}\right) = \frac{23}{17\sqrt{2}}$$

$$S_{\triangle ACF} = \frac{AC \cdot CF \cdot \sin \angle ACF}{2} = \frac{23\sqrt{2} \cdot 34 \cdot 23}{17\sqrt{2} \cdot 2} = 23^2 = 529$$

Ответ: 529

ПИСЬМЕННАЯ РАБОТА

4

Реш

а) Обозначим за $A_1 B_1 C_1$ сечение
треугольного угла плоскостью,
касательной к сфере, $\perp SO_1$ и
имеющей площадь 9

$A_2 B_2 C_2$ - площадью 16

$SO_1 \perp$ плоскость $A_1 B_1 C_1 = O_1$

$SO \perp$ плоскость $A_2 B_2 C_2 = O_2$

Обозначим радиус сферы за r

$$O_1 O = O O_2 = O K = O L = O M = r$$

$$A_1 C_1 B_1 \parallel A_2 C_2 B_2$$

$$\Delta A_1 C_1 B_1 \sim \Delta A_2 C_2 B_2$$

$$\frac{S_{\Delta A_1 B_1 C_1}}{S_{\Delta A_2 B_2 C_2}} = \frac{SO_1^2}{SO_2^2} = \frac{9}{16}$$

$$O_2 O_1 = 2r$$

$$\frac{SO_1}{SO_2} = \frac{3}{4}$$

$$\Rightarrow SO_1 = 6r$$

$$SO_2 = 8r$$

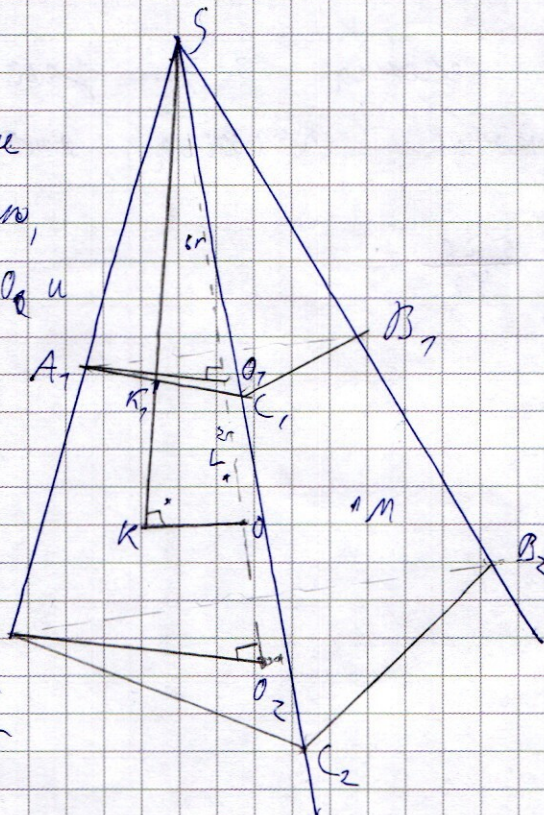
$$SO = 2r$$

$$\sin \angle KSO = \frac{OK}{SO} = \frac{r}{7}$$

$$\cos \angle KSO = \frac{\sqrt{48}}{7}$$

$$\text{Ответ: } \angle KSO = \arcsin\left(\frac{1}{7}\right)$$

$$\delta) SK \cap A_1 C_1 = K_1; \text{ по 2 углам } \Delta O_1 K_1 S \sim \Delta KOS$$



$$\frac{SK}{OS} = \frac{OS}{SK_1}$$

$$SK = \sqrt{48} \cdot r$$

$$SK_1 = \frac{6r - r}{\sqrt{48}} = \frac{5r}{\sqrt{48}}$$

$$\frac{SK}{SK_1} = \frac{8}{r}$$

Площадь сечения 3-х углов плоскостью, проходящей
через точки касания это S_{ABC} , $\left(\frac{SK}{SK_1}\right)^2 = \frac{9 \cdot 8^2}{r^2}$

Ответ: $\frac{576}{49}$