

# МОСКОВСКИЙ ФИЗИКО-ТЕХНИЧЕСКИЙ ИНСТИТУТ

## ОЛИМПИАДА "ФИЗТЕХ" ПО МАТЕМАТИКЕ

### 10 класс

ВАРИАНТ 4

ШИФ

Бланк задания должен быть вложен в р  
Работы без вложенного задания не проверяются.

1. [4 балла] Найдите количество восьмизначных чисел, произведение цифр которых равно 4900. Ответ необходимо представить в виде целого числа.
2. [4 балла] Данна геометрическая прогрессия  $b_1, b_2, \dots, b_{3000}$ , все члены которой положительны, а их сумма равна  $S$ . Известно, что если все её члены с номерами, кратными 3 (т.е.  $b_3, b_6, \dots, b_{3000}$ ), увеличить в 40 раз, сумма  $S$  увеличится в 5 раз. А как изменится  $S$ , если все её члены, стоящие на чётных местах (т.е.  $b_2, b_4, \dots, b_{3000}$ ), увеличить в 3 раза?
3. [4 балла] Решите уравнение  $\left(\frac{x}{2\sqrt{2}} + \frac{5\sqrt{2}}{2}\right) \sqrt{x^3 - 64x + 200} = x^2 + 6x - 40$ .
4. [6 баллов] Решите неравенство  $4x^4 + x^2 + 4x - 5x^2|x+2| + 4 \geq 0$ .
5. [5 баллов] Вокруг крючка с червяком в одной плоскости с ним по двум окружностям плавают карась и пескарь. В указанной плоскости введена прямоугольная система координат, в которой крючок (общий центр окружностей) находится в точке  $(0; 0)$ . В начальный момент времени карась и пескарь находятся в точках  $M_0(-1; 2\sqrt{2})$  и  $N_0(2; -4\sqrt{2})$  соответственно. Скорость карася в два с половиной раза больше скорости пескаря, оба двигаются по часовой стрелке. Определите координаты всех положений пескаря, при которых расстояние между рыбами будет кратчайшим.
6. [6 баллов] а) Две окружности одинакового радиуса 13 пересекаются в точках  $A$  и  $B$ . На первой окружности выбрана точка  $C$ , а на второй – точка  $D$ . Оказалось, что точка  $B$  лежит на отрезке  $CD$ , а  $\angle CAD = 90^\circ$ . На перпендикуляре к  $CD$ , проходящем через точку  $B$ , выбрана точка  $F$  так, что  $BF = BD$  (точки  $A$  и  $F$  расположены по одну сторону от прямой  $CD$ ). Найдите длину отрезка  $CF$ . б) Пусть дополнительно известно, что  $BC = 10$ . Найдите площадь треугольника  $ACF$ .
7. [6 баллов] Найдите все значения параметра  $a$ , при каждом из которых система

$$\begin{cases} |y+x+8| + |y-x+8| = 16, \\ (|x|-15)^2 + (|y|-8)^2 = a \end{cases}$$

имеет ровно два решения.



## ПИСЬМЕННАЯ РАБОТА

№1.

$4900 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \cdot 7$  - эти шесть цифр будут присутствовать в записи восьмизначного числа или  $4900 = 4 \cdot 5 \cdot 5 \cdot 7 \cdot 7$ , тогда все оставшиеся цифры будут единицами.

Количество восьмизнач. чисел:  $8! + 8! = 2 \cdot 912 \cdot 7 \cdot 8 = 80640$  чисел

$$\begin{array}{r} \times 1440 \\ \times 56 \\ \hline 8640 \\ 720 \\ \hline 80640 \end{array}$$

Ответ: 80640

№2

$b_1, b_2, \dots, b_{3000}$  -geom. прогр.; сумма -  $S = \frac{b_1(1-q^{3000})}{1-q} = \frac{b_1(1-q^{3000})}{1-q}$

Члены, ~~крайних~~ начерченные продолжат  $\Rightarrow 1000$

$$q^1 = q^3, \text{ т.к. } \frac{b_6}{b_3} = q^3. \text{ Но условие } S' = 55 = \frac{b_1(1-q^{3000})}{1-q}$$

$$= \frac{b_1 \cdot q^2 \cdot (1-q^{3000})}{1-q^3} \Rightarrow \frac{5 \cdot b_1(1-q^{3000})}{1-q} = \frac{b_1 \cdot q^2 \cdot (1-q^{3000})}{1-q^3}$$

По условию, ~~55~~ сумма новых ~~продолжения~~ чисел

$$\text{равна } S' = 55 = S + 39 \cdot \frac{b_1(1-q^{3000})}{1-q^3} \Rightarrow 4 \cdot \frac{b_1(1-q^{3000})}{1-q} = \frac{39 \cdot b_1 \cdot q^2}{1-q^3}$$

$$\cdot \frac{(1-q^{3000})}{1-q^3} \Rightarrow 4 - 4q^3 = 39q^2 - 39q^3$$

$$35q^3 - 35q^2 - 4 = 0$$

$$35q^3 - 35q^2 - 4q + 4 = 0$$

$$35q^2(q-1) - 4(q-1)(q+1) = 0$$

$$(q-1)(35q^2 - 4q - 4) = 0$$

$q=1$  - не удовл.  
условию.

$$35q^2 - 4q - 4 = 0$$

$$\Delta = K^2 - ac = 12^2 > 0$$

$$q_{1,2} = \frac{2 \pm 12}{35} = \begin{cases} \frac{14}{35} = \frac{2}{5} \\ -\frac{10}{35} = -\frac{2}{7} \end{cases}$$

- не удовл упр, тк  
все члены  $> 0$

Пусть  $S_1$  - сумма при убывающих чеков на  $i$ й месяц

$$S_1 = S + 2 \cdot \frac{b_2(1-q^{3000})}{1-q^2} =$$

$$= \frac{b_1(1-q^{3000})(1+q)}{1-q^2} + 2b_1 \cdot q(1-q^{3000}) - \frac{b_1 q(1-q^{3000})(1+3q)}{1-q^2} =$$

$$= \frac{b_1(1-q^{3000})}{1-q} \cdot \left( \frac{1+3q}{1+q} \right) \Rightarrow \frac{S_2}{S} = \frac{1+3q}{1+q}$$

$$\frac{1+3q}{1+q} = \frac{1+\frac{14}{35}}{1+\frac{14}{35}} = \frac{49}{49} = \frac{11}{7} = \frac{11}{7} = \frac{11}{7} - \text{увеличение } S$$

Ответ:  $S$  увеличился в  $1\frac{4}{7}$  раза

№ 3. № 4.

$$4x^4 + x^2 + 4x - 5x^2(x+2) + 4 \geq 0$$

$$4x^4 + x^2 + 4x - 5x^3 - 10x^2 + 4 \geq 0$$

$$4x^4 - 5x^3 - 9x^2 + 4x + 4 \geq 0 \quad (1)$$

$$x+2 \leq 0$$

$$4x^4 + 5x^3 + 11x^2 + 4x + 4 \geq 0 \quad (2)$$

$$(1) \left\{ \begin{array}{l} 4x^4 - 5x^3 - 9x^2 + 4x + 4 \geq 0 \\ x \leq -2 \end{array} \right. \quad (2) \quad \left\{ \begin{array}{l} x \leq -2 \\ 4x^4 + 5x^3 + 11x^2 + 4x + 4 \geq 0 \end{array} \right.$$

$$x \geq -2$$

## ПИСЬМЕННАЯ РАБОТА

$$4x^4 - 5x^3 - 9x^2 + 4x + 4 \leq 0$$

$$x = -1 \quad 4+4+5-9-4+4=0$$

$$\begin{array}{r} 4 \quad -5 \quad -9 \quad 4 \quad 4 \\ -1 \quad | 4 \quad -9 \quad 0 \quad 4 \quad 0 \end{array}$$

$$(x+1)(4x^3 - 9x^2 + 4) = 0$$

$$x = -1 \quad 3 \cdot (32 - 36 + 4) = 0$$

$$\begin{array}{r} 4 \quad -9 \quad 0 \quad 4 \\ 2 \quad | 4 \quad -1 \quad -2 \quad 0 \end{array}$$

$$\frac{4x^2 - x - 2}{4x^2 - x - 2} = 0 \quad (x+1)(x-2)(4x^2 - x - 2) = 0$$

$$D = 1 + 32 = 33 > 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{33}}{8}$$

$$(x+1)(x-2)\left(x - \frac{1+\sqrt{33}}{8}\right)\left(x - \frac{1-\sqrt{33}}{8}\right) \geq 0$$

$$x \geq -2$$

~~$$\begin{array}{ccccccc} + & - & + & - & + \\ \hline -2 & -1 & \frac{1-\sqrt{33}}{8} & \frac{1+\sqrt{33}}{8} & 2 \end{array}$$~~

$$x \in [-2; -1] \cup \left[\frac{1-\sqrt{33}}{8}; \frac{1+\sqrt{33}}{8}\right] \cup [2; +\infty)$$

$$\begin{bmatrix} (1) \\ (2) \end{bmatrix} \Leftrightarrow x \in (-\infty, -1] \cup \left[\frac{1-\sqrt{33}}{8}; \frac{1+\sqrt{33}}{8}\right] \cup [2; +\infty)$$

$$\text{Ответ: } x \in (-\infty, -1] \cup \left[\frac{1-\sqrt{33}}{8}; \frac{1+\sqrt{33}}{8}\right] \cup [2; +\infty)$$

№3.

$$\left( \frac{x + \sqrt{2}}{2\sqrt{2}} + \frac{5\sqrt{2}}{2} \right) \cdot \sqrt{x^3 - 64x + 200} = x^2 + 6x - 40$$

$$\frac{(x+10) \cdot \sqrt{2}}{4} \cdot \sqrt{x^3 - 64x + 200} = (x+3)^2 - 7^2$$

$$\sqrt{x^3 - 64x + 200} = \frac{4(x+10)(x-4)}{(x+10) \cdot \sqrt{2}}$$

$$x^3 - 64x + 200 = 8x^2 - 64x + 128$$

$$x^3 - 8x^2 + 72 = 0$$

$$x^3 - 6x^2 - 2x^2 + 72 = 0$$

$$x^2(x-6) - 2(x-6)(x+6) = 0$$

$$(x-6)(x^2 - 2x - 12) = 0$$

$$x=6 \quad \text{или} \quad x^2 - 2x - 12 = 0$$

$$D = 13 > 0$$

$$x_{1,2} = 1 \pm \sqrt{13}$$

В) Проверка: 1)  $6^3 - 64 \cdot 6 + 200 = 216 - 384 + 200 = 32 > 0$  - удовл.

$$\begin{aligned} 2) (1 + \sqrt{13})^3 - 64(1 + \sqrt{13}) + 200 &= (1 + \sqrt{13})(1 + 2\sqrt{13} + 13 - 64) + 200 = \\ &= (1 + \sqrt{13})(2\sqrt{13} - 50) + 200 = -48\sqrt{13} + 176 = 16(11 - 3\sqrt{13}) > 0 - \\ &\quad (\text{так как } 11 > 3\sqrt{13} \Rightarrow 11 > 3\sqrt{13}) \end{aligned}$$

$$\begin{aligned} 3) (1 - \sqrt{13})^3 - 64(1 - \sqrt{13}) + 200 &= (1 - \sqrt{13})(-50 - 2\sqrt{13}) + 200 = \\ &= -24 + 48\sqrt{13} + 200 = \cancel{-24} + 48\sqrt{13} = \cancel{16}(\cancel{4}) > 0 - \text{удовл.} \end{aligned}$$

Ответы:  $1 - \sqrt{13}; 1 + \sqrt{13}; 6$

№4.

$$|y+x+8| + |y-x+8| = 16 \quad (1)$$

$$(|x+15|^2 + (|y|-8)^2) = a \quad (2)$$

$$(1) \quad |y+x+8| + |y-x+8| = 16$$

## **ПИСЬМЕННАЯ РАБОТА**

$$\begin{array}{l} \text{1) } \left\{ \begin{array}{l} y = 0 \\ y + x + 8 \geq 0 \\ y - x + 8 \geq 0 \end{array} \right. \end{array}$$

$$2) \quad \left\{ \begin{array}{l} x = 8 \\ y + x - 8 \geq 0 \\ y - x + 8 < 0 \end{array} \right.$$

$$\begin{cases} x = -8 \\ y + x + 8 < 0 \\ y - x + 8 \geq 0 \end{cases}$$

$$4) \begin{cases} y = -16 \\ y + x + 8 < 0 \\ y - x + 8 < 0 \end{cases}$$

$$\begin{cases} y=0 \\ x \in [8, 8] \end{cases}$$

$$2) \begin{cases} x = 8 \\ y \in [-16, 0) \end{cases}$$

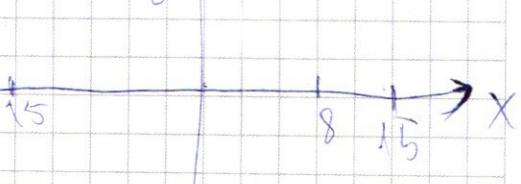
$$3) \begin{cases} x = -8 \\ y \in [-16; 0) \end{cases}$$

$$y) \quad \left\{ \begin{array}{l} y = -16 \\ x = 0 \end{array} \right.$$

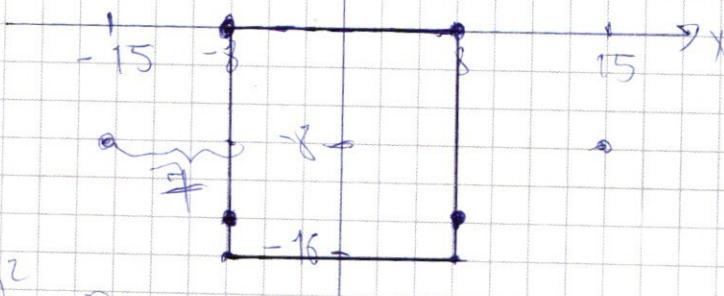
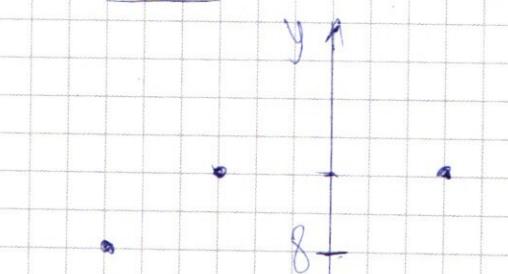
$$x^2 - 3y^2 + x + 2y + 6 = 0$$

$$x^2 - 30x + (289 - a) = 0$$

$$D = k^2 - \alpha = 225 - 289 + \alpha = (\alpha - 64) \cdot 59^{15}$$



~~W~~ ~~1~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ ~~138~~ ~~139~~ 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$$\sqrt{6} \quad AC^2 - AD^2 = (BC + BD)^2$$

$$BD^2 = \frac{FD^2}{2}$$

$$BC^2 = FC^2 - BD^2$$

$$\therefore AC^2 + AD^2 = FC^2 + FD^2 \Rightarrow BG \perp BD$$

$$1) \begin{cases} y=0 \\ y+x+8 \geq 0 \\ y-x+8 \geq 0 \end{cases}$$

$$2) \begin{cases} x=8 \\ y+x+8 \geq 0 \\ y-x+8 \leq 0 \end{cases}$$

$$3) \begin{cases} x=-8 \\ y+x+8 < 0 \\ y-x+8 \geq 0 \end{cases}$$

$$4) \begin{cases} y=+16 \\ y+x+8 \leq 0 \\ y-x+8 < 0 \end{cases}$$
  

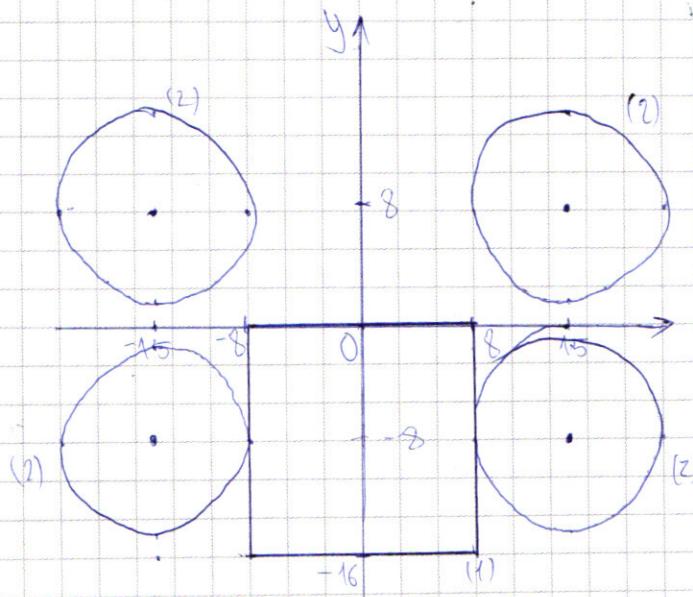
$$\begin{cases} y=0 \\ x \in [-8, 8] \end{cases}$$

$$\begin{cases} x=8 \\ y \in [-16; 0] \end{cases}$$

$$\begin{cases} x=-8 \\ y \in [16; 0] \end{cases}$$

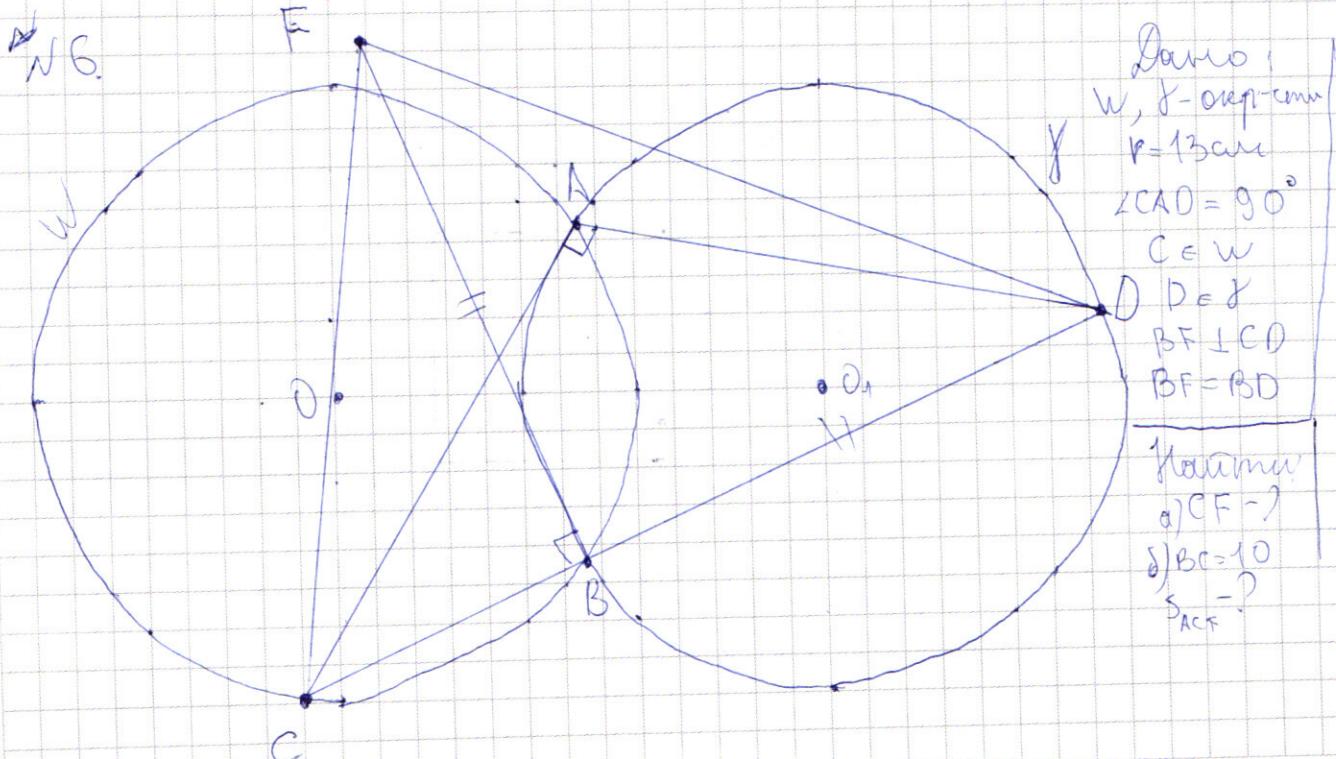
$$\begin{cases} y=-16 \\ x \in (-8; 8) \end{cases}$$

(2)  $(|x|-15)^2 + (|y|-8)^2 = a$  - четырехлистник с радиусом  $10\sqrt{2}$  и  
вершинами в точках  $(\pm 15; 8)$ ;  $(-15; 8)$ ;  $(15; -8)$ ;  $(-15; -8)$



Система имеет два решения, еще где лишил  
ные вершины четырехлистника  
с центром в  $x$   
радиусе равен  $15-8=7$   
 $\Rightarrow a=7^2=49$

Ответ:  $a=49$





## ПИСЬМЕННАЯ РАБОТА

№2

$b_1, b_2, \dots, b_{3000}$  - геом. прогр. ;  $S$ -сумма =  $\frac{b_1 \cdot (1 - q^{3000})}{1 - q}$

~~$S_1 = S; S_2 = 5 \cdot S$~~   $5S = \frac{b_1 \cdot (1 - q^{3000})}{1 - q} + 3g \cdot b_3 \cdot (1 - q^{2000}) =$

$= b_1(1 - q^n)(1 + q) + 3g \cdot b_1 \cdot q^2 (1 - q^{2n}) = b_1(1 - q^n)(1 + q + 3gq^2 + 3gq^{2+n})$

~~$(1 + q^n)$~~  =  $b_1(1 - q^n)(1 + q + 3gq^2 + 3gq^{2+n}) = \frac{5b_1(1 - q^n)}{1 - q}$

$5b_1(1 - q^n)(1 + q) = b_1(1 - q)(1 + q + 3gq^2 + 3gq^{2+n})$

~~$5 + 5q = 1 + q + 3gq^2 + 3gq^{2+n}$~~

~~$3gq^{n+2} + 3gq^3 - 4q - 4 = 0$~~

$S = \frac{b_1(1 - q^{3000})}{1 - q}; 5S = \frac{b_1(1 - q^{3000})}{1 - q} + 3g \cdot b_3 \cdot (1 - q^{3000}) =$

~~$= b_1(1 - q^{1000})(1 + q^{1000} + q^{2000})(1 + q) + 3g \cdot b_1 \cdot q^2 (1 - q^{1000})(1 + q^{1000})$~~

~~$= \frac{b_1(1 - q^{1000})(((1 + q^{1000} + q^{2000})(1 + q) + 3gq^2 (1 + q^{1000}))}{1 - q^2} = 5 \frac{b_1(1 - q^{3000})}{1 - q}$~~

~~$5b_1(1 - q^{1000})(1 + q^{1000} + q^{2000})(1 - q)(1 + q) = (1 - q) \cdot b_1 \cdot (1 - q^{1000})(1 + q^{1000} + q^{2000})(1 + q) + 3gq^2 (1 + q^{1000})$~~

~~$5(1 + q^{1000} + q^{2000})(1 + q) = (1 + q^{1000} + q^{2000})(1 + q) + 3gq^2 (1 + q^{1000})$~~

~~$(4 + 4q)(1 + q^{1000} + q^{2000}) = 3gq^2 + 3gq^{1000}$~~

$$S_2 + 2 \cdot 6 = S + 3g \cdot \frac{b_1 \cdot q^2 (1 - q^{3000})}{1 - q^3}$$

$$\frac{4b_1(1 - q^{3000})}{1 - q} = \frac{3g b_1 \cdot q^2 (1 - q^{3000})}{1 - q^3} \quad 4g = 3q - 3g - 4$$

$$4(1 - q^3) = 3gq^2(1 - q)$$

$$4 - 4q^3 = 3gq^2 - 3gq^3$$

$$4b_1q^3 - 3gq^2 + 4 = 0, \quad q = \frac{b_1 \cdot (1 - q^{3000})(1 + q)}{1 - q^2}$$

$$4b_1q^3 - 3gq^2 + 4 = 0$$

$$35q^3 - 35q^2 + 4q^2 + 4 = 0$$

$$35q^3 - 35q^2 + 4q^2 + 4 = 0$$

$$35q^2(q - 1) - 4(q - 1)(q + 1) = 0$$

$$(q - 1)(35q^2 - 4q - 4) = 0$$

$$q = 1 - \text{не удовл} \quad 35q^2 - 4q - 4 = 0$$

$$D = K^2 - ac = 14^2 - 140 = 17^2 > 0$$

$$q_{1,2} = \frac{2 \pm 12}{35} = \left\{ \begin{array}{l} \frac{14}{35} \\ -\frac{10}{35} \end{array} \right\} \quad -\frac{10}{35} - \text{не удовл.}$$

$$S_2 = \frac{b_1(1 - q^{3000})(1 + 3q)}{1 - q^2} = \frac{b_1(1 - q^{3000}) \cdot (1 + 3q)}{1 - q}$$

$$\frac{1+3q}{1-q} = \frac{1+3 \cdot \frac{14}{35}}{1+\frac{14}{35}} = \frac{\frac{49}{35} + \frac{42}{35}}{\frac{49}{35} + \frac{14}{35}} = \frac{154}{85} = 3 \frac{4}{49} = 3 \frac{4}{7}$$

№3.

$$\frac{(x+5)\sqrt{2}}{252} \cdot \frac{(x+10)\sqrt{2}}{4} \cdot \sqrt{x^3 - 64x + 200} = x^2 + 6x - 40$$

$$x^3 - 64x + 200 \geq 0$$

$$x^3 - 16x - 48x + 200 \geq 0$$

$$(x-4)(x+4) - 8(x-25) \sqrt{x^3 - 64x + 200} =$$

$$x^3 - 100x + 36x + 200 \quad x^3 - 64x + 200 = (x-4)^2 \cdot 8$$

$$\frac{(x+3)^2 - 7^2 \cdot 4}{(x+10) \cdot 8 \cdot \sqrt{2}} = \frac{(x-4) \cdot 2 \cdot \sqrt{2}}{(x-4) \cdot 2 \cdot \sqrt{2}}$$

## ПИСЬМЕННАЯ РАБОТА

N 4

$$4x^4 + x^2 + 4x - 5x^2(x+2) + 4 \geq 0$$

$$\left\{ \begin{array}{l} 4x^4 + x^2 + 4x - 5x^3 - 10x^2 \geq 0 \\ x+2 \geq 0 \end{array} \right.$$

$$4x^4 - 5x^3 - 9x^2 + 4x + 4 \geq 0 \quad (1)$$

$$x \geq -2$$

$$\left\{ \begin{array}{l} 4x^4 + x^2 + 4x + 5x^3 + 10x^2 + 4 \geq 0 \\ x+2 \leq 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} 4x^4 + 5x^3 + 11x^2 + 4x + 4 \geq 0 \\ x \leq -2 \end{array} \right. \quad (2)$$

$$(1) \quad x = -1$$

$$\underline{\underline{4 + 5 - 9 - 4 + 4 = 0}}$$

$$\begin{array}{r} & 4 & -5 & -9 & 4 & 4 \\ -1 & | & 4 & -9 & 0 & 4 & 0 \end{array}$$

$$4x^3 - 9x^2 + 4 = 0$$

$$(x=2)$$

$$\begin{array}{r} & 4 & -9 & 0 & 4 \\ 2 & | & 4 & -1 & -2 & 0 \end{array}$$

$$4x^2 - x - 2 = 0$$

$$D = 1 + 32 = \sqrt{33} > 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{33}}{8}$$

$$\begin{array}{r} & 1 & -\sqrt{33} & 0 & 1 & \sqrt{33} & 2 \\ -2 & -1 & \frac{1-\sqrt{33}}{8} & \frac{1+\sqrt{33}}{8} & 2 \end{array}$$

$$(x+1)(x-2)\left(x - \left(\frac{1+\sqrt{33}}{8}\right)\right)\left(x - \left(\frac{1-\sqrt{33}}{8}\right)\right) > 0$$

$$1) \quad x \in [-2; -1] \cup \left[\frac{1-\sqrt{33}}{8}; \frac{1+\sqrt{33}}{8}\right] \cup (2, +\infty)$$

черновик  чистовик

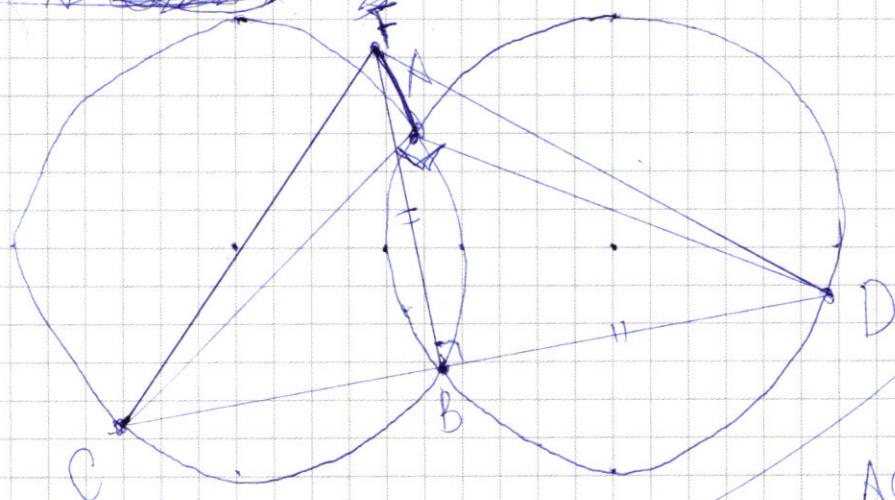
(Поставьте галочку в нужном поле)

~~18~~ ~~17~~ ~~16~~

$$x^3 - 64x + 200 = 8x^2 - 64x + 128$$

$$\therefore x^3 - 8x^2 + 72 = 0 \Leftrightarrow x^3 - 6x^2 - 2x^2 + 72 = 0$$

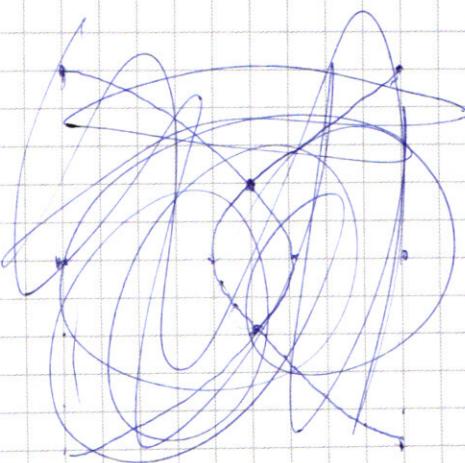
$$\therefore \cancel{x^3 - 6x^2 - 2x^2 + 72 = 0} \quad R = 13$$



$$x^3 - 64x + 200$$

$$AC^2 + AD^2 = CD^2$$

$$AB^2 + DE^2 = DC^2 + BF^2$$



$$x^3(x-6) - 2(x-6)(x+6) = 0$$

$$(x-6)(x^2 - 2x - 12) = 0$$

$$x = 6 \quad x^2 - 2x - 12 = 0$$

$$D = 13 \Rightarrow$$

$$x_{1,2} = \frac{1 \pm \sqrt{13}}{2}$$

$$x^3 - 64 \cdot 6^3 - 64 \cdot 6 + 200 \geq 0$$

$$(1 + \sqrt{13})^3 - 64(1 + \sqrt{13}) + 200 = (1 + \sqrt{13})(1 + 2\sqrt{13} + 13 - 64) + 200 = \\ = (1 + \sqrt{13})(2\sqrt{13} - 5) + 200 = -48\sqrt{13} - 24 + 200 = \\ = 176 - 48\sqrt{13} = 16(11 - 3\sqrt{13}) > 0 \geq 0$$

$$121 > 144$$

$$|y+x+8| + |y-x+8| = 16$$

$$(|x+15|^2 + |y-8|^2) = 0$$